EXPERIMENTAL INVESTIGATION OF FREE SURFACE DYNAMICS AND PRESSURE FLUCTUATIONS IN A CLOSED-CONDUIT HYDRAULIC JUMP

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ABSTRACT

A hydraulic jump may occur in a closed conduit such as shaft spillways, culverts, etc. when the upstream super-critical flow meets a downstream sub-critical flow. If the sub-critical conjugate depth is less than the height of the conduit, a free surface will be present, causing the formation of a hydraulic jump, similar to those formed in open-channel flows. However, if the downstream sub-critical conjugate depth exceeds the closed conduit height, the flow may fill the conduit completely resulting in a pressurized hydraulic jump known as a closed-conduit jump. These jumps are characterized by large vertical surges and low frequency pressure fluctuations that lead to significant vibrations and transient hydraulic loads, resulting in the destabilization of hydraulic structures. While hydraulic jumps in open-channel flows have received extensive attention in the research fraternity, comparatively less information is available on the characteristics of turbulence, pressure fluctuations and energy dissipation in closed-conduit jumps. Consequently, this paper aims to present new experimental data on the flow field and pressure fluctuations associated with such hydraulic jumps in view of their relevance on the stability of hydraulic structures. The present experiments are concerned with closed-conduit jumps in a regular rectangular duct. Free surface fluctuations, if present, were measured using a high speed camera synchronized with a series of pressure transducers mounted on the conduit bed. Water surface dynamics and pressure fluctuations are investigated as a matter of concern in the design of stilling basins and other hydraulic structures. The experimental results show that high pressure fluctuations persist at the downstream end of closed-conduit jumps. Moreover, the mean depth calculated from pressure measurements is smaller than that deduced from the water surface profile obtained using high-speed images, probably due to the presence of air in the flow.

Keywords: Hydraulic jump; closed conduit; water surface profile; pressure fluctuation.

1 INTRODUCTION

A hydraulic jump is categorized by a rapid transition from super-critical to sub-critical flows in natural rivers or man-made hydraulic structures. It exhibits sharp discontinuity in the water surface generating recirculating rollers that entrain a mixture of water and air. It has a strong energy dissipative mechanism due to the formation of high intensity turbulence in a shear zone layer that forms between the roller and the underflowing jet. Interactions between the mean velocity gradients and high-level turbulence produce high pressure fluctuations and forces at the boundary of the hydraulic structures (Schiebe, 1971). Neglecting the dynamic pressure fluctuations at the bed can lead to catastrophic structural failures through different mechanisms such as vibrations, structural resonance and fatigue. In addition, damages due to cavitation also are important in cases where the minimum local pressure fall below the saturation vapor pressure.

Except for the air model study of Rouse et al. (1959), experimental investigation of pressure fluctuations and turbulence in hydraulic jumps have been nearly absent until about 1965. Attempts in the measurement of turbulent velocity fields has encountered many difficulties in the presence of bubbles because bubbles adversely affect the hot-film and optical and acoustic methods of flow measurements. In addition pressure measurements at high Froude number hydraulic jumps were not feasible due to the absence of pressure transducers with a high sampling rate and data acquisition systems capable of recording and analyzing large amount of fluctuating pressure with high accuracy in the pre-mid 1960’s era.

Vasiliev and Bukreyev (1967) could be the first study that attempted measurement of pressure fluctuations by using strain gauge-type transducers within the hydraulic jump in an open-channel flow. They found that the frequency distribution significantly differs from the Gaussian distribution in the initial part of the jump but they are close to each other at the downstream end of the jump. Schiebe (1971) studied the statistical characteristics of the fluctuating pressures exerted on the channel bed under the hydraulic jump. His measurements showed that the pressure distribution is highly skewed and peaked near the toe; and follows the Gaussian distribution in the remaining part of the jump. His results further showed that the maximum root mean square (rms) of pressure fluctuations on the bed occurred at approximately the middle part of the jump and the large pressure fluctuations diminished near its end.
While statistical analyses of the pressure fluctuations in open channel hydraulic jumps have been the subject of numerous researches (Fiorotto and Rinaldo, 1992; Tosato and Bowers, 1988; Vasiliev and Bukreyev, 1967), very few studies were conducted to investigate the pressure distribution in closed-conduit hydraulic jumps. These jumps occur when the sub-critical conjugate depth exceeds the height of the closed conduit and the free surface flow changes to a pressurized flow downstream of the jump. In a closed-conduit hydraulic jump, higher bed pressure fluctuations are observed (Smith and Chen, 1989) and the air bubbles entrained by the jump rise to the conduit ceiling to form air pockets that may cause possible oscillations and structural damage (Ervine and Himmo, 1984).

Lane and Kindsvater (1938) may be the first to study closed-conduit hydraulic jumps. Later Kalinske and Robertson (1943) and Fassò (1956) studied air entrainment in closed-conduit hydraulic jumps and Haindl (1957) showed that the energy loss in closed-conduit hydraulic jumps are smaller and at most equal to the Borda-Carnot loss in hydraulic jumps with a free surface at identical Froude numbers. Rajaratnam (1965) and Smith and Chen (1989) also formulated the hydraulic jump conjugate depth in horizontal and sloped closed-conduit flows (Eqs. [2-3]). Although information on air entrainment and conjugate depth are comparatively more abundant in the literature, there is at yet little information on the flow surface dynamics (Mouaze et al., 2005) and its association with pressure fluctuations on the bed.

Hitherto, most researches in this topic have been conducted with the aim of determining the conjugate depth ratio, energy dissipation and jump length. However none of these studies are concerned with the pressure fluctuations in closed-conduit jumps and how it is linked to the water surface profile. Therefore the current study aims to investigate these issues with simultaneous measurements of the water surface profile and pressure distribution on the bottom of the hydraulic jump. This is very important because closed-conduit jumps can entrain air into the flow under pressure while in open channel hydraulic jumps; the air is released in the roller section (Haindl, 1957).

2 EXPERIMENTAL SETUP

The present experiments were performed in a horizontal rectangular closed conduit that is 5 m long, 0.15 m deep and 20 cm wide. The conduit had glass-sided walls and the bed was half steel and half glass to accommodate flush mounting of the pressure transducers along the centerline of the channel and to provide optical access for the laser sheet. Water was supplied to the head tank from a sump via a centrifugal pump and the discharge was measured by using a magnetic flow meter. In an effort to provide a smooth and uniform inflow, a sluice gate was positioned 1 m downstream of the conduit entrance. The intensity and location of the jump were controlled with the sluice and tail gates, with the latter located at the downstream end of the flume. The sluice gate had a machined streamline lip so that a supercritical stream with a thickness equal to the gate opening was produced.

The test area was located at a distance of 1 m from the sluice gate. Ten pressure transducers were mounted along the centerline of the conduit on the steel portion of the bed. The distance between two successive pressure transducers was 0.1 m (Fig. 1). The location of the toe of the jump was controlled by using the tail gate to ensure that the jump always formed within the test section. The gate opening was constant (3 cm) in all the experiments and the toe of the jump was located more than 1 m from the sluice gate to ensure that the flow is fully developed in the test section. This distance exceeds 50 times the gate opening.

Free surface elevations were measured by using a monochrome high speed camera with a 50-mm lens. Image distortion is kept to a minimum and the entire hydraulic jump from its toe to the end of the jump is kept within view of the camera in the experiment. Four LED lamps were used to illuminate the flow in the background and a thin translucent acrylic sheet was placed between the lamps and glass wall to evenly distribute the light in the background. The test section of the conduit and the camera were surrounded by opaque curtains mounted on a structure; this arrangement ensures that only the desired light is allowed for field illumination.

Since the high speed camera and pressure transducers were connected to the same data acquisition and triggering system, data measurement could commence at the same time. Consequently, simultaneous measurements of the pressure and water surface profiles were carried out for all experiments. The sampling rate of both the high speed camera and pressure transducers was 100 Hz with a frame size of 1920×504 pixels and the measurement duration was 20 seconds. Each monochrome frame of the recorded video was processed in the image processing toolbox in Matlab for extraction of the water surface profile. The image was first subtracted from a reference image taken from the empty conduit. Then, by using the threshold value, the water surface was identified, digitized and scaled from the image. This digitized water surface profile was used for further analyses. Figure 2 shows the adopted procedure in a sample image.
Figure 1. Sketch of the experimental setup (not to scale).

Figure 2. Extraction of water surface profile for Run 2 ($Fr_1 = 2.42$) on the left and Run 6 ($Fr_1 = 3.14$) on the right columns (Flow from left to right).
Table 1 shows the experimental data obtained in this study. The data comprise 16 runs with the same sluice gate opening, resulting in a constant inflow depth of around 0.03 m and conjugate depths varying from 0.06-0.184 m. Both the open-channel (Runs 1-10) and closed-conduit (Runs 11-16) hydraulic jumps were formed in the tests. The super-critical Froude number varies from 1.77-4.59, indicating the formation of both weak and oscillating jumps (Te Chow, 1959). It must be stated that the flow depths in Table 1 are extracted from pressure measurements on the bed. The conjugate depths for Runs 11-16 are therefore computed using the measured pressure by assuming a hydrostatic pressure distribution. Figure 3 shows the measured profiles for all 16 test runs.

### Table 1. Experimental conditions.

<table>
<thead>
<tr>
<th>Run</th>
<th>Q(m³/s)</th>
<th>y₁</th>
<th>V₁</th>
<th>Fr₁</th>
<th>y₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0060</td>
<td>0.031</td>
<td>0.97</td>
<td>1.77</td>
<td>0.055</td>
</tr>
<tr>
<td>2</td>
<td>0.0080</td>
<td>0.030</td>
<td>1.32</td>
<td>2.42</td>
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<tr>
<td>3</td>
<td>0.0071</td>
<td>0.033</td>
<td>1.08</td>
<td>1.91</td>
<td>0.075</td>
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<tr>
<td>4</td>
<td>0.0097</td>
<td>0.032</td>
<td>1.51</td>
<td>2.70</td>
<td>0.102</td>
</tr>
<tr>
<td>5</td>
<td>0.0100</td>
<td>0.031</td>
<td>1.62</td>
<td>2.96</td>
<td>0.117</td>
</tr>
<tr>
<td>6</td>
<td>0.0106</td>
<td>0.031</td>
<td>1.72</td>
<td>3.14</td>
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</tr>
<tr>
<td>7</td>
<td>0.0112</td>
<td>0.031</td>
<td>1.81</td>
<td>3.29</td>
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<tr>
<td>8</td>
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<td>0.032</td>
<td>1.85</td>
<td>3.31</td>
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</tr>
<tr>
<td>9</td>
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</tr>
<tr>
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<td>0.034</td>
<td>1.96</td>
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<tr>
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<td>0.032</td>
<td>2.17</td>
<td>3.88</td>
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<tr>
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<tr>
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<tr>
<td>15</td>
<td>0.0164</td>
<td>0.032</td>
<td>2.57</td>
<td>4.59</td>
<td>0.184</td>
</tr>
<tr>
<td>16</td>
<td>0.0170</td>
<td>0.033</td>
<td>2.57</td>
<td>4.53</td>
<td>0.183</td>
</tr>
</tbody>
</table>

**Figure 3.** Dimensionless longitudinal mean pressure distribution in the streamwise direction.
Each of the y-values in Fig. 3 represents the mean flow depths computed from the time-averaged pressure measurements. It must be noted that all the pressure transducers were first calibrated in a stagnant water tank before the test to ensure accuracy. The horizontal axis in this figure shows the horizontal distance x from the toe of the jump, thus making the toe to be the reference position. Moreover, the data in both axes are rendered dimensionless by using the super-critical flow depth, y. The conjugate depth y, in Table 1 is taken from the most downstream pressure transducer where the water surface showed minimum changes in the streamwise direction.

3 Results

Using the momentum equation and assuming hydrostatic pressure and uniform velocity, Bélanger (1938) derived the well-known simple hydraulic jump equation to compute the conjugate depth ratio for steady open-channel flows in a horizontal rectangular conduit

\[ \frac{y_2}{y_1} = \frac{1}{2} \left( \sqrt{1 + 8Fr_1^2} - 1 \right) \]  

where \( Fr_1 = V_1 / \sqrt{g y_1} \) is the upstream Froude number, g is the gravitational acceleration and \( y_1, y_2 \) and D are previously defined in Fig. 1. Subsequent researchers have proposed equations for the computation of the conjugate depth ratio for closed-conduit hydraulic jumps as follows:

Rajaratnam (1965)

\[ \frac{y_2}{y_1} = \left( \frac{y_1}{D} \right)^n + \left( \frac{y_2}{D} \right)^n + \frac{n+1}{n} \left( 1 + \theta - \left( \frac{y_1}{D} \right)^n \right) \frac{(D - y_1)^n}{(y_1)^n} \]  

\( \theta = 0.0066(Fr_1 - 1)^{14}, \quad n = 2 \)  

Smith and Chen (1989)

\[ \frac{y_2 - y_1}{D} = Fr_1^2 \left( \frac{y_1}{D} \right)^2 \left( 1 - \frac{y_1}{D} \right) + \frac{1}{2} \frac{y_1}{D} \left( \frac{y_1}{D} + D - 2 \right) \]  

Lowe et al. (2011)

\[ \frac{y_2}{D} = \frac{1}{2} + \left( Fr_1^2 + \frac{1}{2} \right) \left( \frac{y_1}{D} \right)^2 - Fr_1^2 \left( \frac{y_1}{D} \right)^3 \]  

Figure 4 shows the comparison between the measured and computed conjugate depths from the current experiments and those computed using Eqs. [1]-[4]. The figure shows that the results computed using the Bélanger equation compare generally well with the experimental data, not only in open-channel but also closed-conduit jumps within the range of \( Fr_1 \) numbers of the study. The results computed from Rajaratnam (1965), Smith and Chen (1989) and Lowe et al. (2011) show some deviations from the measured data.

Energy dissipation in terms of head loss across a simple hydraulic jump can be expressed by

\[ \Delta H = \frac{(y_2 - y_1)^2}{4y_1y_2} \]  

Fig. 5 shows the comparison of the computed results and measured data. The figure clearly reveals that deviations of the computed head loss using Eq. (5) from the measured data for closed-conduit jumps are more significant compared to the conjugate depths. More specifically, the results show that energy dissipation associated with a closed-conduit jump is less than that with an open-channel jump with the same Froude number.
Figure 4. Comparison of measured and computed conjugate depths.

Figure 5. Comparison of measured and computed head loss using Eq. [5].
3.1 Pressure

Figure 6 shows the longitudinal distribution of pressure from three hydraulic jumps with Froude numbers, \( F_r = 1.77, 3.29 \) and 4.53. The maximum and minimum curves, which envelope the mean values, represent, respectively, the highest or lowest pressure deviations from the mean value. The data show that such deviations generally increase with Froude number. Another noteworthy observation is that negative pressure on the bed is only present in jumps with higher Froude numbers (Runs 7-16). Moreover, the magnitude of the negative pressure increases with \( F_r \), an occurrence that may encourage boundary layer separation beneath the jump, as was hypothesized by (Schiebe, 1971).

![Figure 6](image1.png)

**Figure 6.** Comparison of maximum, mean and minimum values of pressure measured underneath open channel (Runs 1 and 7) and closed-conduit (Run 16) hydraulic jumps.

Figure 7 shows variations of the standard deviation \( \sigma \) of the pressure signal beneath the hydraulic jump normalized by a reference value \( \sigma_0 \) measured in still water in the streamwise direction. In open-channel (Runs 1-10) and weak closed-conduit hydraulic jumps (Run 11), the standard deviation and hence pressure fluctuations are observed to decrease from its peak value at the middle part to the end of the jump. At the downstream end of the jump, the pressure fluctuation is very small. On the other hand, the data in Fig. 7 reveal that the standard deviation of the pressure associated with closed-conduit jumps (Runs 12-16) diminishes much slower towards the downstream end of the jump and the persistence of large pressure fluctuations likely will have a strong implication on the design and linings of hydraulic structures in that region. To-date, there has been no study to determine to what extent these pressure fluctuations continue to exist beyond a closed-conduit jump.

![Figure 7](image2.png)

**Figure 7.** Variation of normalized standard deviation of pressure in the stream wise direction.
3.2 Water surface profile and pressure distribution

Figure 8 shows the comparison between the maximum, mean and minimum values of flow depths y at the same locations obtained from using the pressure transducers and high speed camera recordings in Runs 7 and 16. The mean depths obtained from the two methods are close to each other, with values from the pressure measurements being, arguably, slightly smaller than those deduced from the images. A possible reason for the deviation could be the reduced water density due to air entrainment at the water surface.

Figure 8. Comparison between flow depths measured by using high-speed camera and pressure transducers in open channel (Run 7) and closed-conduit (Run 16) hydraulic jumps.

In the open channel hydraulic jump (Run 7), the minimum and maximum values of flow depth are similar to each other at locations upstream of the toe and at the end of the hydraulic jump, but they deviate significantly in the mid-section of the jump.

In the closed-conduit hydraulic jump (Run 16), the free surface exists in a short distance in the upstream section before pressurized flow fills the whole conduit and flow depths can only be compared in 4 stations. The flow depths deduced from the pressure transducers show negative values on the minimum curve. Additionally, the magnitude of peak maximum pressures is more than two times larger than that extracted from the images in this run.

The water surface profiles and pressure distributions in a hydraulic jump are generally considered to be identical to each other if the hydrostatic pressure distribution persists. In this study, although the mean values of pressure and water depth are comparable at each other at relatively small Froude numbers, their fluctuations are not and it is postulated that this phenomenon should be thoroughly investigated in hydraulic jump studies. This is because the occurrence of pressure fluctuations is a result of turbulence and the water surface fluctuations appear to be due to the formation of small waves observed at the jump surface.

4 CONCLUSIONS

The pressure fluctuations and water surface profiles were simultaneously measured in both open channel and closed-conduit hydraulic jumps in a horizontal flume. The comparison of the experimentally measured conjugate depths and energy loss with those computed using published equations reveal small deviations in closed-conduit hydraulic jumps. An examination of the streamwise variation of the standard deviation of pressure shows that the pressure fluctuations with closed-conduit jumps remain high even at the downstream end of the jump. This is in contrast to open-channel hydraulic jumps in which the pressure fluctuations generally would have reduced to a small amount. Simultaneous measurements of the water surface profile and pressure fluctuations showed that the mean depth computed from the pressure transducers is smaller than that deduced from the images. This is likely due to the less-dense aerated flow region at the surface of the hydraulic jump. In addition, larger pressure fluctuations were observed in comparison to water surface fluctuations. The result of this study shows that values of the water surface and pressure may not be simply considered identical and should be studied separately in the context of a hydraulic jump, particularly in a closed-conduit hydraulic jump.

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